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Number Sense and Operations

Number Sense

Classify real numbers as members of one or more subsets: natural, whole, integers, rational, or irrational numbers.

In order to understand the real number system, it is helpful to start by looking at some of the number systems that are subsets of the real number system. To begin, the numbers that are used for counting

1, 2, 3, 4, 5, 6, 7, 8,

are referred to as counting numbers or natural numbers. This set is also called the set of positive integers. When the number 0 is included along with the natural numbers, the set called the whole numbers is obtained:

0, 1, 2, 3, 4, 5, 6, 7, 8,

The numbers in the following set:

-1, -2, -3, -4, -5, -6, -7, -8,

are called the negative integers. When the positive integers, negative integers, and 0 are put together, a set of numbers called integers is obtained:

.....-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5,...

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A rational number is a number that can be written as the ratio of two integers, with, of course, the denominator not equal to 0. All of the integers are rational, since, for example, 7 can be written as $\frac{7}{1}$. The rational numbers also include the fractions, decimals, and percents. Numbers that cannot be written as the ratio of two integers are called irrational numbers. The most common irrational numbers are radical expressions, such as $\sqrt{5}$, $-\sqrt{12}$, $\sqrt[3]{15}$, and so on. Another common irrational number is the number symbolized by the letter π , which is used in geometry to represent the ratio of the circumference to the diameter of a circle. Finally, when the rational and irrational numbers are combined, the resulting set is called the real numbers.

Example:

List all of the subsets of the real numbers (natural numbers, whole numbers, integers, rational, irrational numbers, and real numbers) that the following numbers belong to: $\frac{2}{3}$, -13 , $\sqrt{81}$, $\sqrt{82}$, and 1.007 .

Solution:

The number $\frac{2}{3}$ is rational and real; the number -13 is an integer, rational, and real; $\sqrt{81} = 9$, and so it is natural, whole, an integer, rational, and real; $\sqrt{82}$ is irrational and real; $1.007 = \frac{1007}{1000}$, and so it is rational and real.



Identify properties of the real number system: commutative, associative, distributive, identity, inverse, and closure.

The four basic operations in the real number system are addition, subtraction, multiplication, and division. Certain of these operations satisfy certain properties of the real number system. To begin, note that when you add two numbers, the sum is the same regardless of the order in which you add them, and that when you multiply two numbers, the product is the same regardless of the order in which you multiply them. Therefore, addition and multiplication satisfy the commutative property. The commutative property of addition can be written as $a + b = b + a$, and the commutative property of multiplication can be written as $a \times b = b \times a$. Note that subtraction and division are not commutative.

Next, if you are adding three numbers, you need to decide which two to add first. You know, however, that whichever two numbers you add first, the overall sum will be the same. There is a similar result for multiplication. Addition and multiplication are said to satisfy the associative property, which states that when adding or multiplying three or more numbers, the way that the numbers are grouped (or associated) makes no difference. Using letters, these properties can be written as $(a + b) + c = a + (b + c)$ and $(ab)c = a(bc)$. Remember that parentheses indicate which computation is to be performed first. Also note that subtraction and division are not associative.

Now, consider the expression $4(5 + 6)$. Since the operation in parentheses is to be performed first, we obtain $4(5 + 6) = 4(11) = 44$. Note that, however, if you “distribute” the 4 to the 5 and 6 before adding, you

obtain the same result: $4(5 + 6) = 4(5) + 4(6) = 20 + 24 = 44$. This result is referred to as the distributive property of multiplication over addition, and is written symbolically as $a(b + c) = ab + ac$. Multiplication also distributes over subtraction, and division distributes over both addition and subtraction.

Note that if 0 is added to any real number, the number is unchanged. The number 0, therefore, is called the identity element for addition. Note that, for every real number a , there is a unique number called $-a$ which, when added to a gives the number 0 (the identity element for addition). The number $-a$ is called the additive inverse of a . In a similar way, the number 1 is called the identity element for multiplication, because if any number is multiplied by 1, the result is unchanged. Note that, for every real number a , there is a unique number called $\frac{1}{a}$ which, when multiplied by a gives the number 1 (the identity element for multiplication). The number $\frac{1}{a}$ is called the multiplicative inverse of a .

Finally, a set of numbers is said to be closed with respect to an operation if, when the operation is applied to any pair of numbers in the set, the result is a number in the set. Note, for example, that the positive integers are closed under addition since the sum of any two positive integers is a positive integer. However, the positive integers are not closed under subtraction since the difference of two positive integers is not necessarily a positive integer. For example, $9 - 11 = -2$, a negative integer.

Example 1:

Which properties are illustrated by the following statements?

- a. $(5 + 7) + 4 = 5 + (7 + 4)$
- b. $3 \times 7 = 7 \times 3$
- c. $5(9 - 3) = (5 \times 9) - (5 \times 3)$

Solution:

- a. Associative property of addition.
- b. Commutative property of multiplication.
- c. Distributive property of multiplication over subtraction.

Example 2:

Classify the following statements as true or false:

- a. The integers are closed under subtraction.
- b. The integers are closed under division.
- c. The rational numbers are closed under subtraction.
- d. The rational numbers are closed under division.

Solution:

- a. True.
 - b. False. For example, $\frac{1}{2}$ is not an integer.
 - c. True.
 - d. True.
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Distinguish between finite and infinite sets of numbers.

A finite set of numbers is a set of numbers that has a finite number of elements. You can think of finite numbers as a set of numbers for which it would be theoretically possible to write down every member if you had enough time. Examples of finite sets of numbers include the set of integers between 1 and 16, all of the integer factors of 27, and the set of the square roots of the integers between 3 and 49.

An infinite set of numbers has an infinite number of elements. Sometimes it is easy to identify an infinite set of numbers. For example, the set of integers is infinite, since there is no largest (or smallest) member. If you thought you had a list of all of the integers, you could just add 1 to the largest number in your list and get another integer that is not on the list. In the same way, the set of multiples of 5 is infinite since there is no largest member of this set either. Sometimes, however, it is a bit more difficult to determine whether a set is infinite or not. Consider, for example, the set of rational numbers between 1 and 2. This may appear to be a finite set since you know the largest and the smallest member. However, this set is infinite because it is not possible to write down every element. Suppose, for example, you had a list that you thought contained all rational numbers between 1 and 2. Put the list in order from smallest to largest, and select two numbers that are next to each other. Add these two numbers together and divide the sum by 2, and you will obtain a number that is rational and yet is not on your list. Therefore, you can never write down all of the rational numbers between 1 and 2, and thus this set is infinite.

Example:

Classify the following sets as finite or infinite.

- a. The set of all prime numbers between 7 and 598.
- b. The set of all real numbers between 1.0123 and 1.0124.
- c. The set of all common multiples of 5 and 7.
- d. The set of all positive integral divisors of 360.

Solution:

- a. Finite.
 - b. Infinite.
 - c. Infinite.
 - d. Finite.
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