

Analysis of Change

Determine slope, x -, and y - intercepts of a linear equation.

A linear equation in two unknowns is an equation in which the two variables appear to the first power, but to no higher powers. For example, $y = 2x + 5$ is a linear equation in the variables x and y . A solution of such a linear equation is a pair of values, one for x and one for y , which, when taken together, solve the equation. Such solutions are usually written as ordered pairs of the form (x, y) . For example, $(0, 5)$, $(1, 7)$, and $(-1, 3)$ are solutions to the above equation. When the solutions of a linear equation are graphed on a coordinate axis, they lie on a straight line.

One key property of a straight line is its slope, which is a numerical representation of the steepness of the line. Specifically, the slope of the line tells you how much the line goes up or goes down as you move horizontally one unit from left to right. For example, if a line has a slope of 2, this means that, as you move one unit horizontally from the left to the right, the line rises two units. Two other important properties of a line are its x - and y - intercepts. The x -intercept is the value of x at the point where the line crosses the x -axis. Similarly, the y -intercept is the value of y at the point where the line crosses the y -axis.

If you have a linear equation, the easiest way to determine the slope of the line it represents is to write it in *slope-intercept* form, $y = mx + b$. The coefficient of x , which is called m in this format, will be the slope of the line. Additionally, the value of the constant b will be the y -intercept, since it is the value of y when $x = 0$. To find the x -intercept, set y equal to 0 in the equation, and solve for x .

Example 1:

Determine slope, x -, and y - intercepts of the equation $y = -5x + 2$.

Solution:

This equation is already in slope-intercept form, so the slope is simply the coefficient of x , which is -5 . The y -intercept is 2. To find the x -intercept, set $y = 0$.

$$0 = -5x + 2 \qquad \text{Subtract 2 from both sides}$$

$$-2 = -5x \qquad \text{Divide both sides by } -5$$

$$x = \frac{2}{5} \qquad \text{Thus, the } x\text{-intercept is } \frac{2}{5}.$$

Example 2:

Determine slope, x -, and y - intercepts of the equation $9x + 3y = 4$.

Solution:

The quickest way to answer the question is to re-write the given equation in slope-intercept form.

$$9x + 3y = 4 \quad \text{Subtract } 9x \text{ from both sides}$$

$$3y = -9x + 4 \quad \text{Divide both sides by 3}$$

$$y = -3x + \frac{4}{3} \quad \text{The slope is } -3, \text{ and the } y\text{-intercept is } \frac{4}{3}.$$

To find the x -intercept, set $y = 0$, and solve for x .

$$0 = -3x + \frac{4}{3} \quad \text{Add } 3x \text{ to both sides}$$

$$3x = \frac{4}{3} \quad \text{Divide both sides by 3}$$

$$x = \frac{4}{9} \quad \text{The } x\text{-intercept is } \frac{4}{9}.$$

**Solve formulas for specified variables.**

If an equation has more than one variable, the equation can be solved for a specified variable by expressing that variable in terms of all of the other variables. In order to do this, simply treat all of the other variables as if they were constants, and solve for the specified variable in the usual manner.

Example 1:

Solve $fx - g = h$ for x .

Solution:

The goal is to re-write the equation so that x appears by itself on one side. In order to do this, treat the variables f , g , and h as if they were constants.

$$fx - g = h \quad \text{Add } g \text{ to both sides}$$

$$fx = h + g \quad \text{Divide both sides by } f$$

$$x = \frac{h + g}{f}$$

Example 2:

Solve $7bx + 6b^2 = 4bx + 10b^2$ for x .

Solution:

$$7bx + 6b^2 = 4bx + 10b^2 \quad \text{Subtract } 6b^2 \text{ from both sides of the equation}$$

$$7bx = 4bx + 4b^2 \quad \text{Subtract } 4bx \text{ from both sides of the equation}$$

$$3bx = 4b^2 \quad \text{Divide both sides by } 3b$$

$$x = \frac{4b^2}{3b} = \frac{4b}{3}$$

Example 3:

Solve $\frac{c}{x} + \frac{d}{4x} = 1$ for x .

Solution:

$$4x \left(\frac{c}{x} + \frac{d}{4x} \right) = 4x(1) \quad \text{Multiply both sides by } 4x$$

$$4c + d = 4x \quad \text{Divide both sides by } 4$$

$$x = \frac{4c + d}{4}$$
