

Chapter Ten

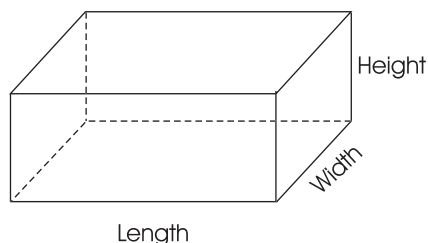
Volume

In chapter eight we spoke of perimeter and circumference, which refer to the lengths of the outer edges of various shapes. In chapter nine we spoke of area, which refers to the space within a shape. This chapter on volume also refers to the concept of space within a shape but, while area applies to two-dimensional figures, volume applies to three-dimensional figures.

What are three-dimensional figures? The mathematical explanation is simply that three-dimensional figures have three dimensions—length, width, and height. Shoe boxes, fish tanks, thermos bottles, rooms—all of these are three-dimensional figures. Even this book is three-dimensional. In real life, most things are. The only things, in fact, that are two-dimensional are things that have only length (also called base) and height, such as the drawings of triangles and circles that we covered in the last chapter.

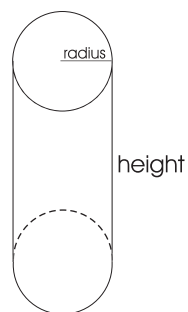
Now let's recall something we learned in the previous chapter on area. Remember that area was expressed in terms of square units, and we had three different ways to write square units. One method used the complete words. A second manner used the abbreviated form of the words, and the third used the number 2

as a superscript (ft.²). Volume will be expressed as cubic units—cubic inches, cubic feet, cubic yards, etc. Again, we will interchangeably use the same three different ways to express cubic units—complete words, abbreviated words, and the number 3 as a superscript (ft.³). Also, like area, volume will require school geometry formulas. In this book, we will cover two different figures: rectangular solids and cylindrical solids.



Rectangular Solid:

$$\text{Volume} = (\text{length})(\text{width})(\text{height})$$



Cylindrical Solid:

$$\text{Volume} = \pi (\text{radius})^2(\text{height})$$

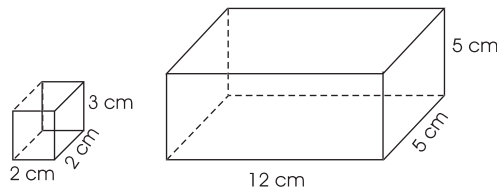
We will want to know how to recognize word problems involving volume when the problem does not use the actual word “volume.” We mentioned a few three-dimensional items such as shoe boxes, fish tanks, and rooms. Each of these items can be filled with things like water, air, or even beach balls. Whenever we are filling a shape with things (even things like water and air), we are dealing with volume.

The two example problems that follow involve volume. In the first, the shape is a rectangular solid, like the picture associated with the first volume formula in

this chapter, and in the second the shape is a cylindrical solid, like the picture associated with the other volume formula. As in the previous chapters on perimeter and circumference and area, whenever we are dealing with geometric shapes, we will always want to have a diagram.

Example: The Quickmart sells small boxes of decorative pebbles for use in flower beds. The small boxes are 2 centimeters long, 2 centimeters wide, and 3 centimeters high. The owner wants to transfer the contents of the small boxes into larger boxes that measure 12 centimeters long by 5 centimeters wide by 5 centimeters high. How many small boxes will be needed to fill one large box?

Let's draw pictures of both sizes of boxes.



We are transferring the contents of small boxes into larger boxes. We are filling boxes with things. This is a volume problem. And since the boxes have the shape of rectangular solids, we will need the volume formula $V = (\text{length})(\text{width})(\text{height})$:

$$\begin{aligned}\text{Volume of small box: } V &= (2 \text{ cm.})(2 \text{ cm.})(3 \text{ cm.}) \\ V &= 12 \text{ cu. cm.}\end{aligned}$$

$$\begin{aligned}\text{Volume of large box: } V &= (12 \text{ cm.})(5 \text{ cm.})(5 \text{ cm.}) \\ V &= 300 \text{ cu. cm.}\end{aligned}$$

To determine how many small boxes of decorative pebbles can be transferred into one large box, we need to see how many times 12 cubic centimeters will fit into 300 cubic centimeters: $300 \div 12 = 25$. The owner of The Quickmart can transfer the contents of 25 small boxes of decorative pebbles into one large box.