

Chapter Twenty-Four

Money Problems

In this chapter, we will learn the special skills or tricks involved in solving word problems related to money. Remember that even though we will not be creating past, present, and future charts as we did in the previous chapter on age problems, everything we learned about writing and solving equations will be helpful in this chapter.

The important thing to keep in mind when faced with problems involving money and coins is to make a distinction between the number of coins (or similar items), and the value of those coins (or similar items).

Let's suppose you have 3 nickels. You must think of this in two different ways. The first is that you have 3 coins and the second is that, because each nickel is worth 5 cents, you have $3 \times 5 = 15$ cents.

You are of course familiar with the indication of money in terms of dollars and cents: \$3.46 is "three dollars and forty-six cents." In order to keep everything uniform, we will use the dollar sign and the decimal point for all monetary values. Instead of writing 15 cents as we did earlier, we will write \$0.15. And

when determining the monetary value of 3 nickels we will multiply $3 \times \$0.05 = \0.15 .

This may seem a bit annoying at first, but it will be worthwhile because it will enable us to easily keep numbers of coins and values of coins separate. During the actual solving of the equations, we will drop the dollar signs but keep the decimal points, because our calculator can easily handle the decimal points.

For each problem in this chapter, we will write two equations. One equation will be based on the number of coins, and the other equation will be based on the value of the coins, as discussed in the previous paragraph. Writing these two equations is the special skill that will enable us to handle these money-based word problems.

Example: Omar has 12 coins that have a total value of \$1.00. If he has only nickels and dimes, how many dimes does he have?

Number of coins: Omar has 12 coins, and they are all nickels or dimes:

$$\text{Nickels} + \text{Dimes} = 12$$

Value of coins: Omar has \$1.00. This is a harder equation to write. We must find a way to express the value of the Omar's nickels and the value of Omar's dimes. Each nickel is worth \$0.05. Omar's nickels are worth $(\$0.05)(\text{Nickels})$. Each dime is worth \$0.10. Omar's dimes are worth $(\$0.10)(\text{Dimes})$:

$$(\$0.05)(\text{Nickels}) + (\$0.10)(\text{Dimes}) = \$1.00$$

As we indicated earlier in this chapter, we will drop the dollar signs but keep the decimal points. If we use the letter N to represent the word Nickels and the letter D to represent the word Dimes, we can rewrite our sentence equations to look like mathematical equations:

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$$N + D = 12$$

$$0.05N + 0.10D = 1.00$$

As we did in chapter twenty-three, “Age Problems,” we have a system of equations. But here, we do not have either “ $N =$ ” or “ $D =$.” We need to take the easier equation, the first equation, and solve it for one of the variables. It does not matter which variable you choose to solve for. We will solve for the first variable by moving the second variable to the other side of the equation:

$$N + D = 12$$

Subtract D from both sides: $N = 12 - D$

Now we have an expression, $12 - D$, which we can substitute for the N in the second equation:

$$0.05(12 - D) + 0.10D = 1.00$$

Distribute the 0.05: $0.6 - 0.05D + 0.10D = 1.00$

$$0.6 + 0.05D = 1.00$$

Subtract 0.6 from both sides: $0.05D = 0.4$

Divide both sides by 0.05: $D = 8$

Omar has 8 dimes.

We can also try the guess-and-check method. Since the total value of Omar’s coins is only \$1.00, we will not want to guess using high numbers of coins.

Perhaps Omar has five dimes. Since he has a total of 12 coins, he must have $12 - 5 = 7$ nickels. The value of these coins would be $(\$0.10)(5 \text{ dimes}) = \0.50 and $(\$0.05)(7 \text{ nickels}) = \0.35 . $\$0.50 + \$0.35 = \$0.85$. This value is not high enough. For our next guess, we will want to choose a larger number of dimes. While this will lower the number of nickels, it will raise the overall value of the coins.